

Problem 9.40

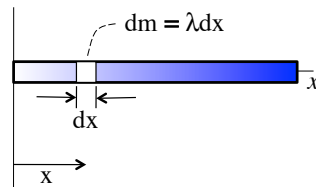
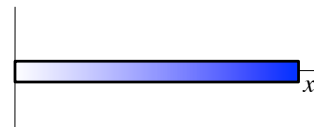
This is a fun *center of mass* problem. You are given the linear density function:

$$\lambda = 50.0 + 20.0x$$

We are trying to derive a relationship for the beam's *center of mass*, given a variable *linear mass density* function. Again, remember what the *center of mass* calculation effectively asks you to do the following: move away from the axis until you find some mass, then multiply that mass by its coordinate. Do this for all masses, add them then divide by the total mass. In this case, we have to start by creating a differentially small amount of mass " dm " comprising a length of rod " dx ," located an arbitrary distance " x " units from the axis. All of that is shown in the sketch.

a.) Determine the total mass in the system.

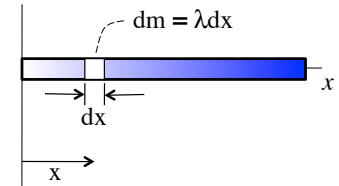
The total mass is just the sum of all the differential masses. That calculation is shown on the next page.



1.)

For the center of mass:

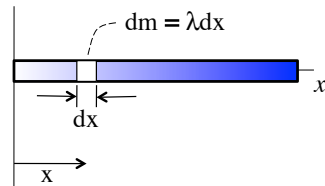
$$\begin{aligned} x_{cm} &= \frac{\int x dm}{m_t} \\ &= \frac{\int_{x=0}^L x (\lambda dx)}{m_t} \\ &= \frac{\int_{x=0}^{L=300m} x (50.0 + 20.0x) dx}{m_t} \\ &= \frac{\int_{x=0}^{L=300m} (50.0x + 20.0x^2) dx}{m_t} \\ &= \frac{\left[50.0 \left(\frac{x^2}{2} \right) + 20.0 \left(\frac{x^3}{3} \right) \right]_{x=0}^{L=300m}}{m_t} \\ &= \frac{25.0 (.300 \text{ m})^2 + 6.67 (.300 \text{ m})^3}{(15.9 \text{ g})} \\ &= .153 \text{ m} \end{aligned}$$



3.)

Noting that λ is in *grams/meter*, the total mass:

$$\begin{aligned} m_t &= \int dm \\ &= \int_{x=0}^{L=300m} \lambda dx \\ &= \int_{x=0}^{L=300m} (50.0 + 20.0x) dx \\ &= \left[50.0x + 20.0 \left(\frac{x^2}{2} \right) \right]_{x=0}^{L=300m} \\ &= 50.0L + 10.0L^2 \\ &= 50.0 (.300 \text{ m}) + 10.0 (.300 \text{ m})^2 \\ &= 15.9 \text{ g} \end{aligned}$$



b.) With the total mass, we can now determine the x -coordinate of the rod's *center of mass* using

$$x_{cm} = \frac{\int x dm}{m_t}$$

2.)